Name: Answer Key Date: _____

The AP Calculus Summer Review
NO CALCULATOR & NO DECIMALS!!!

Let $f(x) = x^2 - 2x + 5$. Find the following:

1. $f(-2)$

$(-2)^2 - 2(-2) + 5$

$4 + 4 + 5$

$\boxed{13}$

2. $f(x+2)$

$(x+2)^2 - 2(x+2) + 5$

$x^2 + 4x + 4 - 2x - 4 + 5$

$\boxed{x^2 + 2x + 5}$

3. $f(x+h)$

$(x+h)^2 - 2(x+h) + 5$

$\boxed{x^2 + 2xh + h^2 - 2x - 2h + 5}$

Use the graph of $f(x)$ to answer the following.

4. $f(0) =$ -4

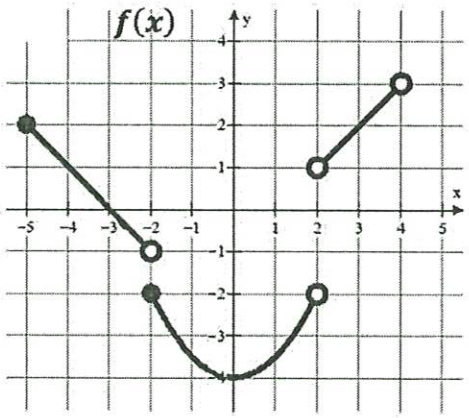
$f(4) =$ DNE / undefined

$f(-1) =$ -3.5

$f(-2) =$ -2

$f(2) =$ DNE / undefined

$f(3) =$ 2



$f(x) = 2$ when $x =$?

$x = 3, -5$

$f(x) = -3$ when $x =$?

$x = -1.5, 1.5$

Write the equation of the line that meets the following criteria. Use slope-intercept $y = mx + b$ or point slope form $y - y_1 = m(x - x_1)$.

5. $m = 3$ and $(4, -2)$

Point Slope:

$y + 2 = 3(x - 4)$

$y = 3x - 14$

$y = mx + b$

$-2 = 3(4) + b$

$-14 = b$

$y = 3x - 14$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$ $(-5, 7)$

Point Slope:

$y - 7 = -\frac{3}{2}(x + 5)$

$y - 7 = -\frac{3}{2}x - \frac{15}{2}$

$y = -\frac{3}{2}x - \frac{1}{2}$

$y = mx + b$

$7 = -\frac{3}{2}(-5) + b$

$7 = \frac{15}{2} + b$

$-\frac{1}{2} = b$

$y = -\frac{3}{2}x - \frac{1}{2}$

7. $f(4) = -8$ and $f(-3) = 12$ $(4, -8)$ $(-3, 12)$

$m = \frac{12 - (-8)}{-3 - 4} = \frac{20}{-7}$

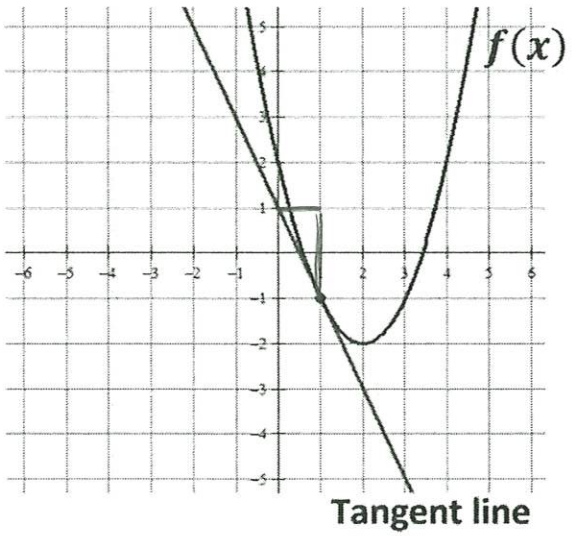
$y + 8 = -\frac{20}{7}(x - 4)$

$y + 8 = -\frac{20}{7}x + \frac{80}{7}$

$y = -\frac{20}{7}x + \frac{24}{7}$

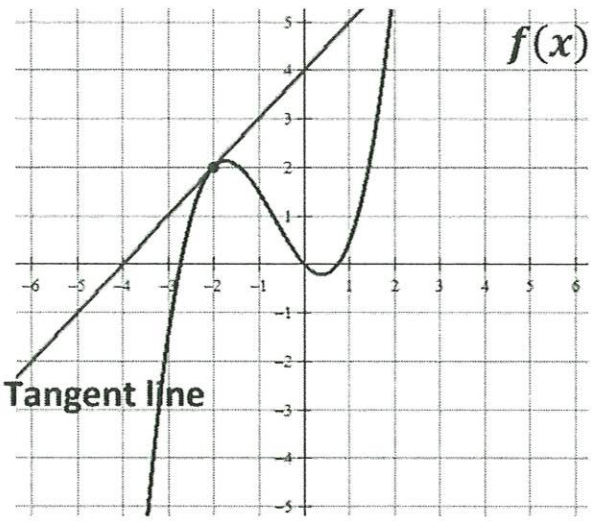
Write the equation of the tangent line shown in slope-intercept form.

8. The line tangent to $f(x)$ at $x = 1$



$y = -2x + 1$

9. The line tangent to $f(x)$ at $x = -2$



$y = x + 4$

Multiple Choice! Remember SLOPE = $\frac{y_2-y_1}{x_2-x_1}$

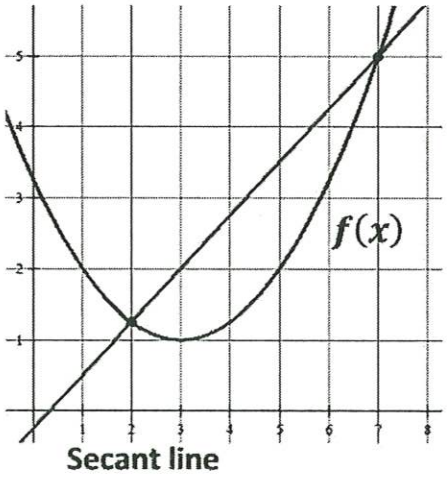
10. Which of the following represents the slope of the secant line?

A) $\dfrac{7-2}{f(7)-f(2)}$

B) $\dfrac{f(7)-2}{7-f(2)}$

C) $\dfrac{7-f(2)}{f(7)-2}$

D) $\dfrac{f(7)-f(2)}{7-2}$



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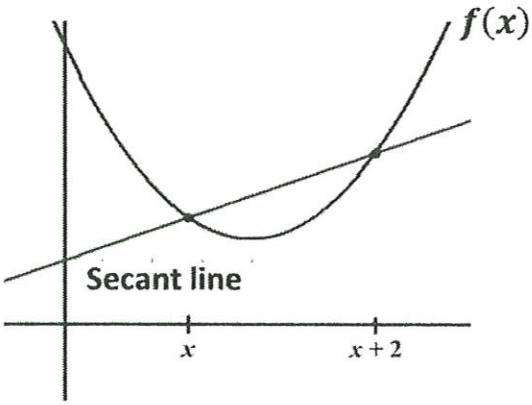
11. Which of the following represents the slope of the secant line?

A) $\dfrac{f(x)-f(x+2)}{x+2-x}$

B) $\dfrac{f(x+2)-f(x)}{x+2-x}$

C) $\dfrac{f(x+2)-f(x)}{x-(x+2)}$

D) $\dfrac{x+2-x}{f(x)-f(x+2)}$



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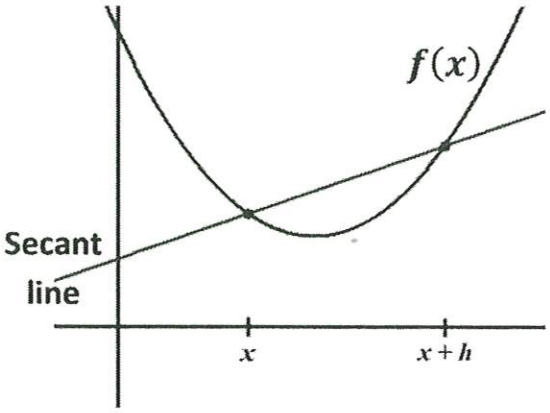
12. Which of the following represents the slope of the secant line?

A) $\dfrac{f(x+h)-f(x)}{x-(x+h)}$

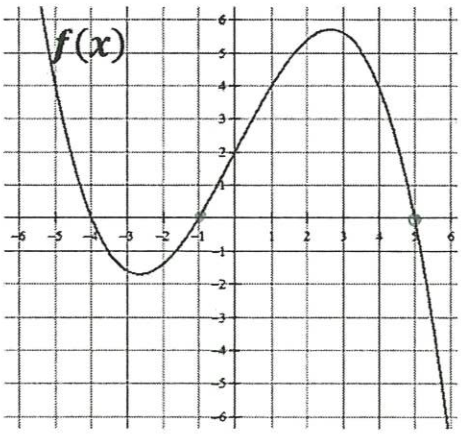
B) $\dfrac{x-(x+h)}{f(x+h)-f(x)}$

C) $\dfrac{f(x+h)-f(x)}{x+h-x}$

D) $\dfrac{f(x)-f(x+h)}{x+h-x}$



13. Which of the following statements about $f(x)$ is true?



I. $f(2) = 0$ X

II. $(x+4)$ is a factor of $f(x)$ ✓ $x = -4$ is a zero.

III. $f(5) = f(-1)$ ✓

A) I only

B) II only

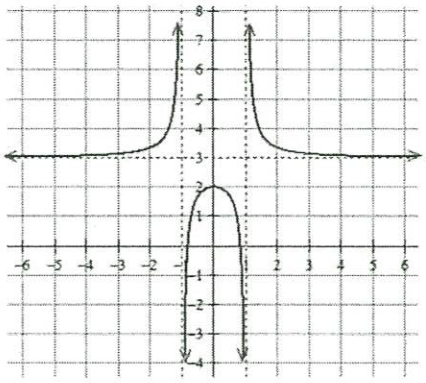
C) III only

D) I and III only

E) II and III only

State the domain and range of the function in interval notation. Find all horizontal and vertical asymptotes.

14.



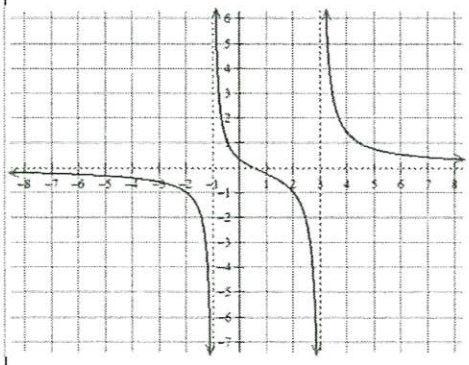
Domain:
$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Range:
$(-\infty, 2] \cup (3, \infty)$

Horizontal Asymptote(s):
$y = 3$

Vertical Asymptote(s):
$x = -1$
$x = 1$

15.



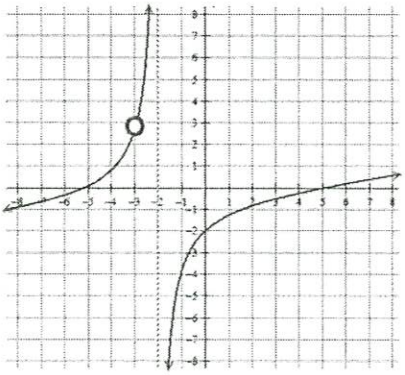
Domain:
$(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

Range:
$(-\infty, \infty)$

Horizontal Asymptote(s):
$y = 0$

Vertical Asymptote(s):
$x = -1$
$x = 3$

16.



Domain:
$(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$

Range:
$(-\infty, \infty)$

Horizontal Asymptote(s):
None

Vertical Asymptote(s):
$x = -2$

Multiple Choice!

17. Which of the following functions has a vertical asymptote at $x = 4$?

A) $\frac{x+5}{x^2-4}$ **B)** $\frac{x^2-16}{x-4}$ $\frac{(x-4)(x+4)}{(x-4)}$ **C)** $\frac{4x}{x+1}$ **D)** $\frac{x+6}{x^2-7x+12}$ $(x-3)(x-4)$

E) None of the above

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18. Consider the function $f(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

$\frac{(x-3)(x-2)}{(x-2)(x+2)}$

I. $f(x)$ has a vertical asymptote of $x = 2$

II. $f(x)$ has a vertical asymptote of $x = -2$ ✓

III. $f(x)$ has a horizontal asymptote of $y = 1$ ✓

A) I only

B) II only

C) I and III only

D) II and III only

E) I, II, and III

Rewrite the following using rational exponents. For example, $\frac{1}{\sqrt[3]{x^2}} = x^{-2/3}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$

$x^{3/5} + (2x)^{1/5}$

20. $\sqrt{x+1}$

$(x+1)^{1/2}$

21. $\frac{1}{\sqrt{x+1}}$

$(x+1)^{-1/2}$

22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$

$x^{-1/2} - 2x^{-1}$

23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$

$(4x^3)^{-1} + \frac{1}{2}x^{3/4}$

$\frac{1}{4}x^{-3} + \frac{1}{2}x^{3/4}$

24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

$\frac{1}{4}x^{-1/2} - 2(x+1)^{-1/2}$

Rewrite each expression in radical form and positive exponents. For example, $x^{-2/3} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-1/2} - x^{3/2}$

$\frac{1}{\sqrt{x}} - \sqrt{x^3}$

26. $\frac{1}{2}x^{-1/2} + x^{-1}$

$\frac{1}{2\sqrt{x}} + \frac{1}{x}$

27. $3x^{-1/2}$

$\frac{3}{\sqrt{x}}$

28. $(x+4)^{-1/2}$

$\frac{1}{\sqrt{x+4}}$

29. $x^{-2} + x^{1/2}$

$\frac{1}{x^2} + \sqrt{x}$

30. $2x^{-2} + \frac{3}{2}x^{-1}$

$\frac{2}{x^2} + \frac{3}{2x}$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$

31. $e^x + 1 = 2$

$\ln(e^x) = \ln 1$

$x = 0$

32. $3e^x + 5 = 8$

$3e^x + 5 = 8$

$3e^x = 3$

$\ln e^x = \ln 1$

$x = 0$

33. $e^{2x} = 1$

$\ln e^{2x} = \ln 1$

$2x = 0$

$x = 0$

34. $\ln x = 0$

$e^{\ln x} = e^0$

$x = 1$

35. $3 - \ln x = 3$

$-\ln x = 0$

$e^{-\ln x} = e^0$

$x = 1$

36. $\ln(3x) = 0$

$e^{\ln(3x)} = e^0$

$3x = 1$

$x = \frac{1}{3}$

37. $x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \quad x = 3$

38. $e^x + xe^x = 0$

$e^x(1 + x) = 0$

$e^x = 0$ NOT possible

$1 + x = 0$

$x = -1$

39. $e^{2x} - e^x = 0$

$e^x(e^x - 1) = 0$

$e^x = 0$ NOT possible

$e^x - 1 = 0$

$\ln e^x = \ln 1$

$x = 0$

You know trig values in radians! You can use the Unit Circle or Special Right Triangles to find the following.

40. $\sin\frac{\pi}{6}$ $\frac{1}{2}$

41. $\cos\frac{\pi}{4}$ $\frac{\sqrt{2}}{2}$

42. $\sin 2\pi$ 0

43. $\tan\pi$ 0

44. $\sec\frac{\pi}{2}$ $\frac{1}{\cos \pi/2} = \frac{1}{0}$ undefined

45. $\cos\frac{\pi}{6}$ $\frac{\sqrt{3}}{2}$

46. $\sin\frac{\pi}{3}$ $\frac{\sqrt{3}}{2}$

47. $\sin\frac{3\pi}{2}$ -1

48. $\tan\frac{\pi}{4}$ 1

49. $\csc\frac{\pi}{2}$ $\frac{1}{\sin \pi/2}$ 1

50. $\sin\pi$ 0

51. $\cos\frac{\pi}{3}$ $\frac{1}{2}$

52. Find x where $0 \le x \le 2\pi$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

53. Find x where $0 \le x \le 2\pi$:

$\tan x = 0$

$x = 0, \pi, 2\pi$

54. Find x where $0 \le x \le 2\pi$:

$\cos x = -1$

$x = \pi$

Solve the following trig equations where $0 \le x \le 2\pi$.

55. $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

56. $\cos x = -1$

$x = \pi$

57. $\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

58. $2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

59. $\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$ $\theta = \pi/6$

$\frac{x}{2} = \frac{\pi}{6}$ $\quad \frac{x}{2} = \frac{11\pi}{6}$

$x = \frac{\pi}{3}$ $\quad x = \frac{11\pi}{3}$ ← NOT in domain interval

61. $\tan x = 0$

$x = 0, \pi, 2\pi$

62. $\sin(2x) = 1$ $\theta = \pi/2$

$2x = \frac{\pi}{2}$

$x = \frac{\pi}{4}$

63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$ $\theta = \pi/3, 2\pi/3$

$\frac{x}{4} = \frac{\pi}{3}$ $\quad \frac{x}{4} = \frac{2\pi}{3}$

$x = \frac{4\pi}{3}$ $\quad x = \frac{8\pi}{3}$

State the domain and range of each function.

Function	*Domain*	*Range*
64. $f(x) = \sqrt{x-4}$	$(-\infty, 4) \cup (4, \infty)$ $x \geq 4$	$(0, \infty)$ $y \geq 0$
65. $g(x) = (x-3)^2$	$(-\infty, \infty)$ $\mathbb{R}$	$(0, \infty)$ $y \geq 0$
66. $y = \ln x$	$(0, \infty)$ $x > 0$	$(-\infty, \infty)$ $\mathbb{R}$
67. $y = e^x$	$(-\infty, \infty)$ $\mathbb{R}$	$(0, \infty)$ $y > 0$
68. $y = \sqrt{4-x^2}$	$[-2, 2]$ $-2 \leq x \leq 2$	$[0, 2]$ $0 \leq y \leq 2$

Simplify.

69. $\frac{\sqrt{x}}{x}$ $\frac{x^{1/2}}{x} = x^{-1/2} = \boxed{\frac{1}{\sqrt{x}}}$

70. $e^{\ln x}$ $\boxed{x}$

71. $e^{1+\ln x}$ $e^1 \cdot e^{\ln x}$ $\boxed{ex}$

72. $\ln 1$ $\boxed{0}$

73. $\ln e^7$ $\boxed{7}$

74. $\log_3 \frac{1}{3}$ $3^x = \frac{1}{3}$ $\boxed{x = -1}$

75. $\log_{1/2} 8$ $(1/2)^x = 8$ $\boxed{x = -3}$

76. $\ln \frac{1}{2}$ *calc $\boxed{\sim 0.693}$

77. $27^{2/3}$ $\sqrt[3]{27^2} = \boxed{9}$

78. $(5a^{2/3})(4a^{3/2})$ $\boxed{20a^{13/6}}$

79. $\frac{4xy^{-2}}{12x^{-1/3}y^{-5}}$ $\frac{x^{4/3}y^3}{3}$ $\boxed{\frac{x^{4/3}y^3}{3}}$

80. $\left(4a^{\frac{5}{3}}\right)^{3/2}$ $\sqrt{4^3} \cdot a^{15/6}$ $\boxed{8a^{5/2}}$

Use the functions defined below to determine each of the following.

$f(x) = \{(3,5),(2,4),(1,7)\}$ $\quad$ $h(x) = \{(3,2),(4,3),(1,6)\}$

$g(x) = \sqrt{x-3}$ $\quad$ $k(x) = x^2 + 5$

81. $(f+h)(1)$

$f(1) + h(1)$

$7 + 6$

$\boxed{13}$

82. $(k-g)(5)$

$(5^2+5) - (\sqrt{5-3})$

$\boxed{30 - \sqrt{2}}$

83. $f(h(3))$

$f(2) = \boxed{4}$

84. $g(k(7))$

$K(7) = 49+5 = 54$

$g(54) = \boxed{\sqrt{51}}$

85. $h(3)$

$\boxed{2}$

86. $g(g(9))$

$g(9) = \sqrt{6}$

$g(\sqrt{6}) = \boxed{\sqrt{\sqrt{6}-3}}$

87. $f^{-1}(4)$

$f^{-1}(x) = \{(5,3),(4,2),(7,1)\}$ $\quad$ $f^{-1}(4) = \boxed{2}$

88. $k^{-1}(x)$

$y = x^2 + 5$

$x = y^2 + 5$

$y = \sqrt{x-5}$

$\boxed{K^{-1}(x) = \sqrt{x-5}}$

89. $k(g(x))$

k

$\sqrt{x-3}$

$(\sqrt{x-3})^2 + 5$

$x - 3 + 5$

$\boxed{x+2}$

90. $g(f(2))$

$f(2) = 4$

$g(4) = \sqrt{4-3} = \sqrt{1} = \boxed{1}$

Fill out the Unit Circle.